GLENTYDE

CAPITAL ADVISORS

ABSTRACT The price of many assets contains a component frequently referred to as the **present value of growth opportunities**. This component of an asset's price is frequently calculated using a very intuitive formula. In this brief article we'll show that this component can be easily justified at an elementary level. Doing so provides a confidence, of course, in the customary manner of determining an asset's PVGO. But perhaps more importantly, it provides a better foundation upon which to build a more robust PVGO model which doesn't rely on the simplifying assumptions inherent in the familiar elementary model.

Whether it's a single share of stock or an entire company, many assets have a value which includes a component frequently called the *present value of growth opportunities* ("*PVGO*"). Although the term is most commonly associated with equity shares, a little thought shows that its underlying concept is actually inherent in nearly any asset whose future value is at least partially contingent upon future reinvestment (or "plowback") decisions. The business enterprise controlled by a small group of co-owners, or whose capital investment decisions are in the hands of a management team, provide the classic scenarios in which PVGO is most frequently encountered and debated. But PVGO is no less important in the context of a securities portfolio, a retirement plan, or a corporate division. In short, if one can think of any asset for which there exists flexibility in adjusting the payout / plowback ratio across time, one has automatically thought of an asset for which PVGO is probably a factor. In addition, when one considers that PVGO sometimes accounts for **more than** 100% of an asset's value (notably, equity shares of a promising start-up with minimal early-stage revenues), PVGO takes on even more prominence.

PVGO is usually computed using a simple subtraction of the hypothetical value of the asset under a "full payout" assumption, from its hypothetical value under an assumption of some partial reinvestment scenario. If this subtraction yields a positive value, of course, it indicates that value is being created as a result of some level of plowback into assets which are clearly earning more than the appropriate opportunity rate of the asset's owners. A negative result indicates value destruction and hence the need to critique the plowback decision.

This paper has no aspirations of becoming the PVGO analysis to end all analyses. We'll keep it brief, we'll stick with an elementary approach to the matter, but by the end we'll have some confidence that the intuitive PVGO calculation mentioned previously does indeed hold up mathematically. Also—and probably of greater importance—a simple examination of what's going on "under the hood" should provide a foundation upon which one could then relax the simplifying assumptions of the elementary approach, and confidently build a superior PVGO model with better real-world relevance.

In particular, the naïve approach simplifies our work in a couple of notable ways:

- A disregard of inflation
- An assumption of steady-state growth in perpetuity

With respect to inflation, we'll stick to working with *nominal* cash flows and *nominal* earnings and discount rates. Since PVGO analysis usually involves a lengthy time horizon, inflation plays a notable role. Feel free to revise the analysis into terms of constant dollars and real rates if you like, but herein we'll rely on the fact that nominal dollars discounted at nominal rates is equivalent to real dollars discounted at real rates, under most assumptions.

The steady-state growth assumption is a real burr in most folks' saddles, but an analogy can be drawn with the argument some people have with M&M's[1] famous *irrelevance* proof: Dealing with the real world means relaxing the simplifying assumptions, but one cannot do so safely and intelligently without *first* obtaining a rigorous *understanding* of the assumptions. The elegant power of M&M's two propositions was in providing practitioners with a clear analysis of the matter, employing certain assumptions. And **that**, ladies and gentlemen, made it possible to develop valid real-world models based on a solid understanding of just what the assumptions meant, and precisely what ramifications the relaxing thereof would impart on the model's results.

In that spirit we'll hang on to our perpetual growth assumption. The ubiquitous intuitive calculation of PVGO calls for it, and besides, by working with a familiar mathematical construct we'll gain a better understanding of how to modify the assumption properly in more sophisticated approaches.

SIDEBAR Most PVGO calculations in practice do adopt a more real-world relevant approach to the growth question. But these are frequently just variations on the steady-state assumption. For example, in many industries it's more or less true that the industry eventually reaches a no-growth point at which any promises of superior returns have been evaporated away by new entrants and by expansion of incumbents, resulting in an equilibrium state. With no positive-NPV opportunities, the firms eventually migrate to a full-payout state and growth converges to zero (in real terms). This scenario is easily handled by using a ***finite*** geometric series to model the pre-equilibrium, positive-growth period, and then a full-payout assumption thereafter.

To facilitate our analysis we'll need a small notation inventory:

- d is a popular choice for the dividend[2] payout amount on a share of stock in these models; who are we to break with tradition?
- We'll assume we're standing at time $t = 0$, and any subscripts will denote a relative point in time. d_n, for example, is the cash payout ***n*** periods hence.
- k and r play the roles, respectively, of the appropriate discount rate / cost of capital, and the rate of return on assets.
- Speaking of assets, a_n will denote the amount of assets during the span of period ***n***.
- Again with a nod to tradition, b will serve as the reinvestment / plowback rate. (In many economic models, the obvious choice ***p*** is usually busy playing *profit*, *probability*, or *price*.)
- The growth rate will be handled by g and also by rb. Their equality is easily seen in a moment.
- Observing that we can economize a bit with our notations, ra_n and $(1 - b)$ will serve, respectively, as net earnings for period *n*, and the payout rate; no need to assign special symbols to these two chaps.

One final observation, and then we're underway: ***k***, ***r***, and ***b*** (and thus (1 – ***b***) and ***g***) are assumed to be constants. Dividend payouts and asset levels will, of course, be time-dependent variables.

First let's knock out that $g \equiv rb$ identity I mentioned a second ago. More specific to the PVGO model, we're interested in verifying that the constant growth rate *g* in the dividend payout stream is equivalent to the product of the assets' return and the reinvestment rate.

The dividend payout at the end of any period *n* is simply the earnings for such period, times the payout rate. In turn, the period's earnings can be disassembled into the product of two factors: the asset level during the period, and the earnings *rate* on such assets. All together,

$$d_n = a_n r(1-b) \qquad (1)$$

The asset level in effect during a period equals the asset level of the previous period, plus any earnings retained at the end of such previous period. Or,

$$a_n = a_{n-1} + a_{n-1} rb = a_{n-1}(1+rb) \qquad (2)$$

This new expression for a_n lets us restate the dividend payout at the end of period *n* in (1) as

$$d_n = a_{n-1}(1+rb)r(1-b) \qquad (3)$$

Now, the growth (or decay) rate *g* in the dividend payout from any arbitrary period to the immediately subsequent period is then given by

$$g = \frac{d_{n+1}}{d_n} - 1 = \frac{a_n r(1+rb)(1-b)}{a_n r(1-b)} - 1 = rb \qquad (4)$$

That little digression in showing the equivalence of *g* and *rb* will prove its worth later. (The sharp-eyed reader has noticed that we've also proven that under the constant earnings rate and plowback assumptions, ***assets*** are also growing at the constant rate *g*. Can you see where, about three equations back?) Now on with the main show.

As previously mentioned the staple PVGO calculation is simply the subtraction of the asset's value under a full payout assumption, from its value assuming some positive amount of plowback:

$$PVGO = \frac{d_1(1-b)}{k-g} - \frac{d_1}{k} \qquad (5)$$

The minuend up there is just the familiar constant-growth model[3] while the subtrahend is the stock's present value as a perpetuity (i.e., full payout, no growth, $d_1 = d_2 = d_3 = \ldots$). Note that the numerator of the growth-model portion of (5) gives the expected payout net of the assumed reinvestment. Also note that (5) values the growth opportunities as of time t_0.

'NUTHER SIDEBAR We spot a chance to put our $g \equiv rb$ identity to good use right away. What value does growth have if earnings are reinvested to earn a rate that's *only equal* to the shareholder's opportunity rate? Intuitively, we'd say zero, zip, zilch. Such growth should neither increase nor decrease value. Let's see if intuition holds up well; if $r = k$, then

$$PVGO = \frac{d_1(1-b)}{k-g} - \frac{d_1}{k} = \frac{d_1(1-b)}{k-rb} - \frac{d_1}{k} = \frac{d_1(1-b)}{k-kb} - \frac{d_1}{k} =$$

$$\frac{d_1(1-b)}{k(1-b)} - \frac{d_1}{k} = \frac{d_1}{k} - \frac{d_1}{k} = \text{zero, zip, zilch}$$

The familiar way expressing PVGO in (5) is stated fairly intuitively. We're first using the constant-growth model to estimate a value assuming that some portion *b* of the earnings are plowed back into assets each period, inducing a growth rate *g* in the underlying dividend-paying "engine", and thus in the stream of payouts as well. From that, we deduct the value we'd expect if all earnings were instead paid out in full, in perpetuity. The difference, positive or negative, should logically represent the value of the reinvestment strategy vs. a full payout strategy.

Sounds logical enough, but does the difference given by (5) actually square up with a present-value scrutiny of the reinvestment strategy itself? Put differently, can we derive (5) a little more rigorously than merely by writing down a formula that *seems* to make sense? That question brings us at last to the article's *raison d'être*: a demonstration that any PVGO implied by the familiar formula (5) does indeed agree with such an analysis.

Suppose first a full-payout situation. Neither assets nor payouts grow or diminish (remember, *r* is a constant in this little world), and so $d_1 = d_2 = d_3 = \ldots$. We'll make it easier on ourselves, then, by dropping the subscript on the payouts from the "base" assets. As we'll see in the next paragraph we'll hold the base assets constant, and so the earnings and the payouts each period will all simply be *d*.

Now we suppose that at t_1 we'll reinvest some portion *b* of the t_1 payout into a new asset. (In reality the plowback is probably commingled into the base assets, but it helps our visualization to think of the reinvestment as creating a separate and distinct asset.) Hence at t_1 this asset stands at *db*.

Over the ensuing period this asset will generate earnings of $dbr = dg$ (there's another use of our previously-proven equality). But if we assume that we'll apply the same reinvestment policy to the earnings on this *new* asset, our first payout at t_2 will be $dg(1 - b)$. We also note that if we continue to apply the same plowback strategy to this new asset indefinitely, this first payout at t_2 represents the first of a constant-growth sequence of payouts. We can therefore value this new asset at t_1 using Gordon…

$$\frac{dg(1-b)}{k-g} \qquad (6)$$

Backing up one period the new asset's t_0 value is…

$$\frac{dg(1-b)}{(k-g)(1+k)} \qquad (7)$$

Again, (7) gives the t_0 value of the single asset created by the t_1 reinvestment of a portion of the earnings from the base assets. We next notice that we'll in effect create an identical asset *each period*, as we reinvest the same portion b of the base assets' earnings. Playing off of (7), the t_0 present value of the assets created at t_2, t_3, t_4, …, are given by

$$\frac{dg(1-b)}{(k-g)(1+k)^2}, \quad \frac{dg(1-b)}{(k-g)(1+k)^3}, \quad \frac{dg(1-b)}{(k-g)(1+k)^4}$$

…and so on. Thus the aggregate t_0 present value of **all** of the reinvestments of the base assets' earnings into perpetuity is the sum of this *geometric series*. We note that this geometric series has an *initial term* and *common ratio*, respectively, of

$$\frac{dg(1-b)}{(k-g)(1+k)}; \quad \frac{1}{1+k}$$

Since the common ratio lies strictly in (–1, 1) the series converges to a finite sum. Remembering the formula for a convergent geometric series with initial term α and common ratio γ is given by $S = a/(1-g)$, we can then give the t_0 aggregate present value of all the reinvestments as

$$\frac{dg(1-b)}{(k-g)(1+k)\left(1-\frac{1}{1+k}\right)} = \frac{dg(1-b)}{k(k-g)} \qquad (8)$$

Almost there, but (8) is missing something. In order to create this sequence of new assets—one each period, and whose total value is given by (8)—we have to invest db of the base assets' earnings each period, beginning at t_1. To get the **net** PV of the growth opportunities in total we'll need to deduct from (8) the present value of the investments themselves, which conveniently form a perpetuity…

$$PVGO = \frac{dg(1-b)}{k(k-g)} - \frac{db}{k} \qquad (9)$$

Now (9) seems to bear a family resemblance to the familiar PVGO expression in (5), but they're not identical twins. Let's see if we can prove their equality:

$$(9) = \frac{dg(1-b)}{k(k-g)} - \frac{db}{k} = \frac{dg(1-b)}{k(k-g)} - \frac{db(k-g)}{k(k-g)} =$$

$$\frac{dg(1-b)-db(k-g)}{k(k-g)} = \frac{d(g-gb-kb+gb)}{k(k-g)} =$$

$$\frac{d(g-kb)}{k(k-g)} = \frac{d(g-kb+k-k)}{k(k-g)} =$$

$$\frac{d(k-kb)-d(k-g)}{k(k-g)} = \frac{dk(1-b)-d(k-g)}{k(k-g)} =$$

$$\frac{dk(1-b)}{k(k-g)} - \frac{d(k-g)}{k(k-g)} = \frac{d(1-b)}{k-g} - \frac{d}{k} = (5) \text{ QED}$$

Not bad. Let's take stock of what we've accomplished. We first considered the rendering of PVGO as it's usually given in (5). We noted it certainly makes sense, as the net PV of the growth opportunities *should* represent the value of the assets assuming some nonzero plowback of earnings, less the assets' value they'd have under a full payout, no-growth assumption.

But to verify for ourselves that (5) does indeed capture the theoretical value of the growth opp'ys, we built from scratch a reinvestment scenario in which some constant portion of the base assets' earnings are rolled into new assets each period. We then, independently of (5), derived a PVGO formula (9) that models this built-from-scratch scenario. And then we wrapped it up by verifying the equality (9) = (5). Cool.

But as mentioned at the outset, this result wasn't our sole objective. While the familiar (5) is a succinct formula for PVGO under the right assumptions, it obscures the activity "under the hood". By re-creating the growth situation from scratch we've hopefully given ourselves a foundation upon which we can begin to relax the assumptions upon which (5) rests, and then modify the model appropriately to handle such issues as inflation and assumed changes in the reinvestment rate across time. I think that the development of PVGO in (9) herein is more tractable for this purpose, providing a blueprint for more sophisticated models than the standard PVGO expression in (5).

Such additional model development can be dealt with at another time, but for now, we've given ourselves a decent starting point.

---

[1] Modigliani, F. and M. H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," *American Economic Review* 48 (June 1958), pp. 261-297.

[2] I freely admit that, despite my earlier insistence that the PVGO concept has much broader application than solely to stock shares, I will resort to using terms (such as *dividend*) which imply a stock share context. This should place the discussion onto more familiar terms for those readers (the majority, certainly) who are accustomed to working with PVGO in such a context. The wider applicability of the concepts across other asset classes is not diminished thereby.

[3] Frequently called the Gordon Growth Model, from its popular articulation in Gordon, M. J., and E. Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," *Management Science* 3 (October 1956), pp. 102-110.